ECCOMAS Thematic Conference on
Computational Methods in Tunnelling (EURO:TUN 2007)
J. Eberhardsteiner et.al. (eds.)
Vienna, Austria, August 27-29, 2007

IDENTIFICATION OF CONSTITUTIVE AND GEOMETRICAL PARAMETERS OF NUMERICAL MODELS WITH APPLICATION IN TUNNELLING

J. Meier[1], M. Datcheva[2] and T. Schanz[1]

[1] Laboratory of Soil Mechanics, Bauhaus- Universität Weimar,
Coudraystrasse 11c, 99421 Weimar, Germany
e-mail: joerg.meier@vint.de ; tom.schanz@uni-weimar.de

[2] Institute of Mechanics, Bulgarian Academy of Sciences,
Acad. G. Bontchev, bl. 4, 1113 Sofia, Bulgaria
datcheva@imbm.bas.bg

Keywords: Back-analysis, 3D finite element modelling, Particle Swarm Approach, Tunnelling, Fault zone.

Abstract. *One of the major challenges by tunnelling is the location and properties assessment of the fault zones existing near the tunnel face. Geological prospecting often gives just a rough draft of the local site conditions. Geophysical investigations on the other hand provide relatively sufficient information in many practical cases but are linked to high costs and additional restrictions on the working design. Supplementary to geotechnical measurement programs, numerical simulations are widely used for planning complex infrastructure projects like tunnels. In this article a concept is presented, how numerical modelling in conjunction with displacement measurements at the tunnel lining over construction time can be used for identification of geometrical and constitutive model parameters of an existing weak zone in front of the tunnel face. Both the host rock mass and the weak (fault) zone are supposed to be composed of materials obeying linear elastic constitutive law. The numerical model built in ABAQUS is used to solve the forward problem and this way to obtain the displacements to be compared with the measured ones in order to perform model identification. A short overview is presented concerning different methods for parameter estimation problem solution. In order to evaluate the ability of the proposed technique to identify the fault zone position, thickness and material parameters a synthetic experiment is performed and used afterwards in a model back analysis. The results reported here show that particle swarm optimization (PSO) strategy with its feature to provide the extremum in the predefined trusted zone and its robustness in nonlinear applications is a promising tool for geometric and material model identification. The prospective of using PSO in solving specific tasks related to tunnelling is also discussed.*

1 INTRODUCTION

For design and construction of a tunnel structure, it is important to know the mechanical properties of surrounding rock masses, the geological conditions and the morphological features of the project site. Only limited number of field tests is usually conducted for such a purpose. These field tests are often costly and time-consuming with dispersive data and mostly covering small test domain. To obtain the overall rock mass properties, many researchers have used the back-analysis method in geotechnical engineering, especially in tunnelling and underground cavern construction. Back analysis as an indirect method can be very helpful for cost-effective site model identification and in the same time for validation of the field test results. Back analysis is generally defined as a technique which can provide the significant system parameters by analyzing its response behaviour. The induced by the excavation displacements of rock masses can be measured easily and reliably at the tunnel walls and because of that the displacement-based back-analysis techniques had been well explored by many research groups (see e.g. [5]). Back analysis problems may be solved in two different ways, defined as inverse and direct approaches (see [2]). The inverse approach solves some of the model parameters based on measured displacements while the direct approach is based on an iterative procedure correcting the trial values of unknown parameters by minimizing error functions. Hence, applying direct approach to the back analysis no formulation of the inverse problem is required and the solution is obtained by combining forward solutions and optimization procedure. Figure 1 shows the flow chart for a direct back-analysis iterative process.

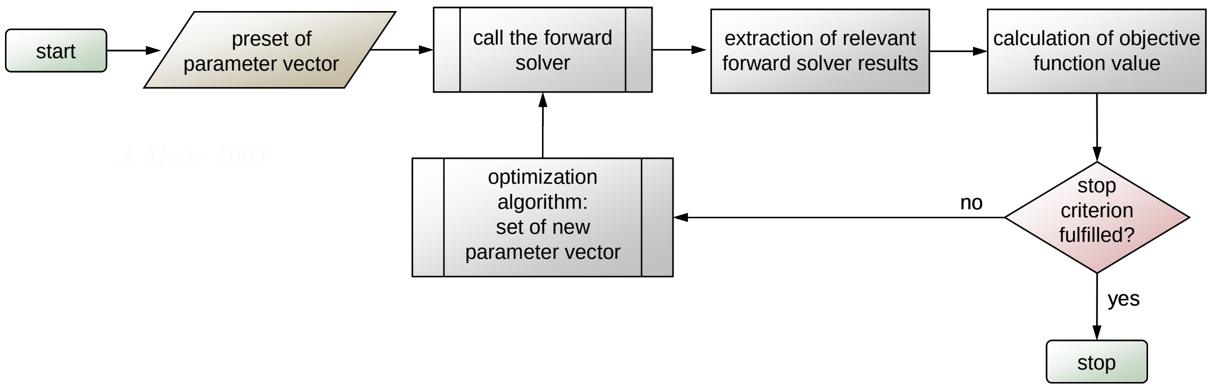

Figure 1: Back analysis flow chart (direct approach).

This paper addresses a direct approach to the back analysis method applied to tunnelling. The objective is to obtain key material and geometrical parameters determining the rock mass behaviour emphasizing on the influence of fault zones ahead of the tunnel face.

2 OPTIMIZATION TECHNIQUE AND SOLVER

The proper choice of the parameter estimation technique and the method for solution of the objective function minimization problem is of a paramount importance for the efficiency and robustness of the back analysis results.

Over the past decade a number of optimization algorithms have been used extensively in optimization tasks, from gradient-based algorithms using continuous and in most cases convex design spaces, to non-gradient probabilistic- based search algorithms widely applied for global and non-convex design exploration. In the latter category fall algorithms that have been developed by mimicking natural phenomena. One can refer to simulated annealing genetic algorithms and evolutionary strategies among others. Recently, a family of optimization algorithms has been developed based on the simulation of social interactions among members of a specific species looking for food sources. From this family of algorithms, the two most promising are ant colony optimization and particle swarm optimization (PSO). The latter is based on the social behaviour reflected in flock of birds, bees, and school of fish, see [3], [4]. There is a vast literature concerning PSO. Short description of the PSO can be found elsewhere and particularly for geotechnical applications in [7] and [8].

At present, optimization of back-analyzed parameters is often carried out by using the method of least square. In this method, the objective function $f(x)$ for more than one time series is defined as:

$$f(x) = \sum_{g} [w_g \cdot f_g'(x)], \tag{1}$$

with

$$f_g'(x) = \frac{1}{m} \sum_{h=1}^{m} w_h \left(y_g^{calc}(x) - y_g^{meas}\right)_h^2 \tag{2}$$

In the above equations $\boldsymbol{x}$ denotes the parameter vector to be estimated and w_g are positive weighting factors associated correspondingly with the error measure $f'_g(x)$. By the weights w_g the different series g could be scaled to the same value range and different precisions could be merged, e. g. a series possessing high precision is included with higher weighting factor compared to more uncertain data. The particular numbers for the weights have to be given manually respecting the engineers experience and they have to be specified depending on the optimization problem. The weighting factors w_h are used to provide a possibility for considering different precision and measurement errors within one and the same data-series. The dimensions of the weighting factors can be taken in the way to obtain dimensionless objective function quantity.

In this work the output least square technique is used for the parameter estimation and the particle swarm optimization (PSO) is the algorithm chosen for minimizing the objective function.

3 NUMERICAL APPLICATION

The investigation performed in [1] shows that a detailed knowledge of the location, size and shape of blocks, as well as the anisotropic matrix fabric, during construction enables safe and economical tunnelling. Interpretation of 3D deformation state and history caused by stepwise excavation is common practice in tunnel constructions. Schubert et al. ([9]) present a literature survey on the displacement monitoring data and proposed a hybrid concept combining evaluation of displacement vector orientations and their matching with the numerical prediction. The deviation of measurements from the numerically obtained response of a model without fault or weak zones enables to derive geometrical and constitutive information regarding those fault zones that are close to the construction. Such an approach provides a tool for qualitative indication of weak or stiff layers ahead the tunnel face. That is why for evaluating the ability of our back analysis procedure we chose as a numerical example to identify the position, width and stiffness of an existing weak zone ahead the tunnelling direction for an idealized tunnel construction and rock environment.

In order to make the results of the back analysis more simply and fast applicable to design, it is often assumed that the materials involved in the model are homogeneous and obey linear elastic constitutive law. This approach especially holds in the case of 3D tasks as considered here. This assumptions is restrictive but also preserving and not the last it gives an opportunity to avoid numerical difficulties in solving the forward problem. Therefore in this example the response of all materials involved follows isotropic a linear elastic stress-strain relationship.

3.1 Model description

Figure 2 depicts the structure of the model with a fault zone located at *L* meters from the model frontal face measured from its top edge and having a model sidewall projection width of *D* meters. It is accepted that the fault dip direction and the angle of incidence θ, are known from an engineering-geology investigation, and in the present model θ equals to 66°.

The excavation has a circular cross section with a radius of 10 meters. The total sizes of the model are 100x100x200 meters. The discretization based on tetrahedral elements used in the 3D ABAQUS finite element model is also shown in Figure 2. Even the model obeys symmetry along axis 1 the whole model is considered in order to exercise future more realistic and therefore more complex tunnelling projects whose model sizes can not be reduced. For the sake of simplicity in this idealized numerical model we neither consider supporting elements as tunnel lining nor details of opening the profile.

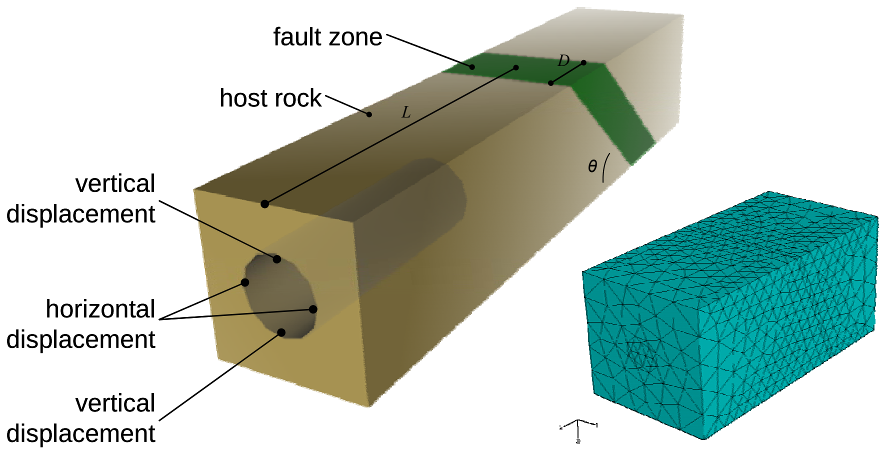

Figure 2: Schematic representation of the numerical experiment and the finite element discretization (ABAQUS model).

The synthetic measurements data are obtained from the numerical solution of the following boundary value problem (BVP). The model geometry is presented in Figure 2 with $L = 120$ m, $D = 6$ m and the material parameters are given in Table 1. The initial stresses are imposed by the geostatic step in ABAQUS with prescribed horizontal to vertical stresses ratio equal to 1. The model in Figure 2 is meant to be a mountain cut-out with overburden height of 500 meters. To account for this model feature, on the top side of the model a pressure is applied that corresponds to the overburden weight and equals to 1.23E+04 kN/m^2. This value is calculated assuming the overburden rock mass to be homogeneous, with a density of 2500 kg/m^3. The displacements history data is collected during the simulation of ten tunnel excavations, each of 10 meters length. Observation points are located at the tunnel crown, bottom and at the side wall, see Figure 2. Thus after completing all the ten excavation steps, the total number of observations points used consequently in the back analysis is 44. At each observation point one measurement is taken and in our case this is a component of the displacement vector, see notation in Figure 2. Before starting the excavation there are 4 observation points and each excavation step adds to the data four new values. The obtained solution of the here described problem is further called measured displacements vector, noted as $\boldsymbol{u}_m$.

	Host rock	Fault zone
E [kN/m^2]	2E+8	5E+7
Poisson ratio	0.2	0.35
ρ [kg/m^3]	2500	2200

Table 1: Material parameters.

3.2 Statistical analysis

For the sake of simplicity in this example it is assumed that the constitutive properties (density, Young modulus and Poisson ratio) of the host rock are known with acceptable accuracy. All uncertainty is with the values of the Young modulus E of the fault zone material as well as the two geometrical characteristics L and D. Thus the parameter vector to be estimated is $\boldsymbol{x}=\{E,L,D\}$. In order to improve and ensure the efficiency of the back analysis it is of significant importance to check if the set of parameters to be identified may be reduced and if for the prescribed trusted zone the optimization problem is well posed. For this purpose a statistical analysis is done based on the results from a performed Monte Carlo procedure including 2000 parameter sets. The objective function is taken following the equations (1) and (2). The different time-series used in the present work are all of the same physical nature and to them the same measurement technique and precision are applicable. Therefore it can be set both $w_g = 1$ and $w_h = 1$. For the example in hand the data series include displacements of a comparable order and therefore all the displacements can be considered to belong to one data series. Thus the objective function takes the form:

$$f(x) = \frac{1}{m}\sum_{h=1}^{m}\left(u_c(x) - u_m\right)_h^2 \qquad (3)$$

The applied constraint intervals are:

$$1E{+}06 \le E \le 1E{+}08 \text{ [kN/ m}^2\text{]};\;\; 119.5 \le L \le 121.5 \text{ [m]};\;\; 5 \le D \le 6.5 \text{ [m]}\;; \qquad (4)$$

Figure 3a shows the principle scheme of the matrix plot used here to visualize the results of the Monte Carlo simulation. A standard mathematical tool for examining multi-dimensional data sets is the scatter plot matrix, see [6]. It is included in the matrix plot present in Figure 3a where each non-diagonal element shows the scatter plots of the respective parameters. Therefore the matrix is symmetric. The matrix element *D-B* for example may suggest that the involved parameters *B* and *D* are not independent but strongly correlated. The matrix plot diagonal elements (*A-A*, …, *D-D*) show plots where the value of the objective function is given over the parameter which is associated with the corresponding column. These plots are called hereafter objective function projections. If the problem is well-posed, each of these plots of the objective function projections has to present one firm extreme value as it is the case in the diagram *D-D*. Otherwise the respective parameter could not be identified reliably. By filtering out data points witch have objective function values larger then a certain threshold level, the distribution of the remaining points gives a rough idea of the size and shape of the extreme value (solution) space. For further statistical analysis the well-known linear 2D correlation coefficient can be calculated from the individual scatter plots. Referring to [10], in the analysis of our tunnel case we accept variables with correlation coefficient less than 0.5 as "non-correlated".

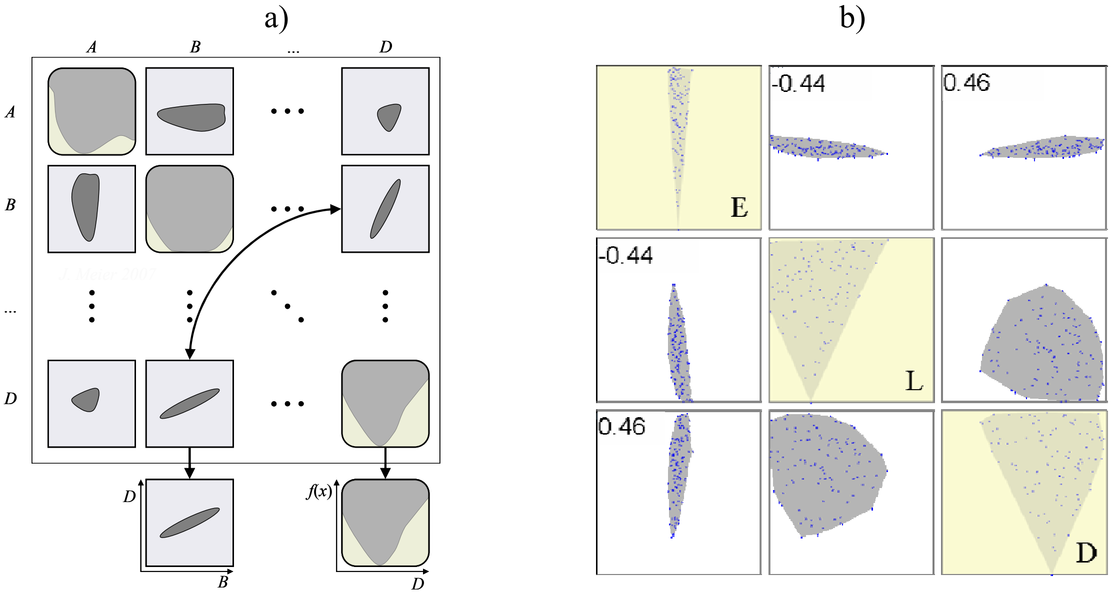

Figure 3: Statistical analysis via matrix plot; a) principle scheme;
b) filtered results of the Monte Carlo simulation done in this work

The matrix plot in Figure 3b shows the results of the Monte Carlo simulation done in this work and it summarizes the best about a hundred runs based on the condition for the objective function value not to exceed 1.0E-03. From the diagonal plots it can be concluded that each of the three parameters *E*, *L* and *D* secures an objective function extremum within the search interval because each plot is well presenting a unique objective function minimum. The cross parameters diagrams in the non-diagonal boxes are projections of the objective function hypersurface on the respective parameter axes planes. These diagrams show the correlation level between respective parameters under the accepted restriction on the objective function to be less than 1.0 E-03. Based on these plots it is concluded that there is no significant correlation between the parameters *E*, *L*, and *D*. The correlation coefficients are given in Figure 3b. The correlation between *D* and *L* lies within the interval [-0.4, 0.4]. Since the correlation coefficients are less than the threshold level of 0.5, the parameters are considered to be non-correlated. Another conclusion concerns the constraint intervals and it is gained from the scatter plots and due to the filtering of the Monte Carlo simulation results used for their representation. There are well pronounced empty areas and this invokes a discussion regarding the appropriateness of the chosen trusted zones. For the purpose of our investigation we assumed the chosen constraint intervals to be satisfactory proper and representative.

3.3 Particle swarm optimization solution

The results of the PSO procedure are discussed in this section. The algorithm of the back analysis (noted hereafter with *BA*) is the following:

Step 1: forward solution of the BVP problem described in section 4 with given *E*, *L*, *D*. (solver ABAQUS)
Output: displacements of the observation points, vector $\boldsymbol{u}_c$ [m].

Step 2: calculate the objective function, equation (3).
Output: measure of the deviation between $\boldsymbol{u}_c$ and $\boldsymbol{u}_m$ [m].

Step 3: solving the constraint optimization problem (particle swarm approach) with constraints given by equation (4)
Output: new set of parameters *E*, *L*, *D* .

Step 4: satisfaction of the optimization criterion – YES = stop, NO = go to Step 5.

Step 5: correcting the input file for the forward calculation (ABAQUS input file) with the output set from Step 3 and go to Step 1.

The relative narrow constraint intervals of the unknown parameters *D* and specially *L* in equation (4) were chosen due to re-meshing and convergence problems in the forward calculations in case of large fault zone's shape and position changes. With an adapted forward calculation this restriction could be surmounted and the constraint intervals can be expanded.

```
'initialization of swarm
Create particle list
For Each Particle
     Set initial particle position and velocity
Next Particle

'processing loop
Do

     'get global best particle position
     Determine best position in past of all particles

     'determine current particle positions
     For Each Particle
          Calculate and set new velocity based on a corresponding
equation
          Calculate and set new position based on a corresponding
equation
     Next Particle

     'parallelized calculation of objective function values
     For Each Particle
          Start forward calculation and calculate objective function
value
     Next Particle

     'join all calculations
     Wait for all particle threads

     'post-processing
     For Each Particle
          If current objective function is less than own best in
past:
               Save position as own best
          End If
     Next Particle

Loop Until Stop Criterion is Met
```

Table 2: Pseudo-code of used particle swarm optimizer.

Table 2 presents the pseudo-code of the implemented PSO used in this work. The computer-program developed by the first author implements the PSO algorithm shown in Table 2. Because of the PSO ability to be parallelized within each cycle of the algorithm, the runs of the forward solver could be done simultaneously.

Figure 4 helps to interpret the performance of the optimization solution. The optimization history plots shown there ensure our confidence in the uniqueness of the solution after the last excavation step. PSO has been performed with 10 individuals. The solution obtained after 25 runs (cycles) of the *BA* scheme is given in Table 3. The deviation reported in Table 3 can be reduced to 0.1% if the optimization solution procedure is run 50 times and this fact is well presented by the results shown in Figure 5. This figure illustrates the objective function and relative parameters optimization range history plots. In Figure 5 the relative parameter x_{rel} (in percents) corresponding to a given component x of the parameter vector $\boldsymbol{x}$ is defined as $100(x-x_{min})/(x_{max}-x_{min})$, where x_{max} and x_{min} are the left and right ends of the accepted for x constraint interval.

	Model	Back analysis	Deviation
E [kN/m^2]	5E+7	4.969E+7	0.031
L [m]	120	120.04	0.04
D [m]	6	5.94	0.06

Table 3: Result of the particle swarm optimization.

4 CONCLUSIONS

A 3D back-analysis procedure is proposed which is based on elastic displacements measured at the tunnelling face immediately after the excavation, least square technique, correlation analysis and PSO algorithm. The ability of the proposed procedure has been demonstrated and discussed on an example for estimating, considering synthetic data, both constitutive and geometrical model parameters. The main outcome is that 3D back-analysis offers a promising tool for gaining information both on material and geometrical features in different geotechnical projects and particularly in tunnelling. For performing the direct back analysis procedure the general purpose finite element code ABAQUS has been coupled with a self developed software tool for PSO. As a conclusion one can state, that with use of the particle swarm algorithm a successful back analysis of the unknown parameters could be done with a justifiable amount of forward calculation runs. Next step should be the application of the proposed approach to real field situations.

ACKNOWLEDGEMENTS

The first author acknowledges the support by the German Research Foundation (DFG) via the projects SCHA 675/7-2 and SCHA 675/11-2 (“Geomechanical modelling of large mountainous slopes”). The second author has been supported by the Bulgarian National Scientific Fund under the grant No TH-1304/03.

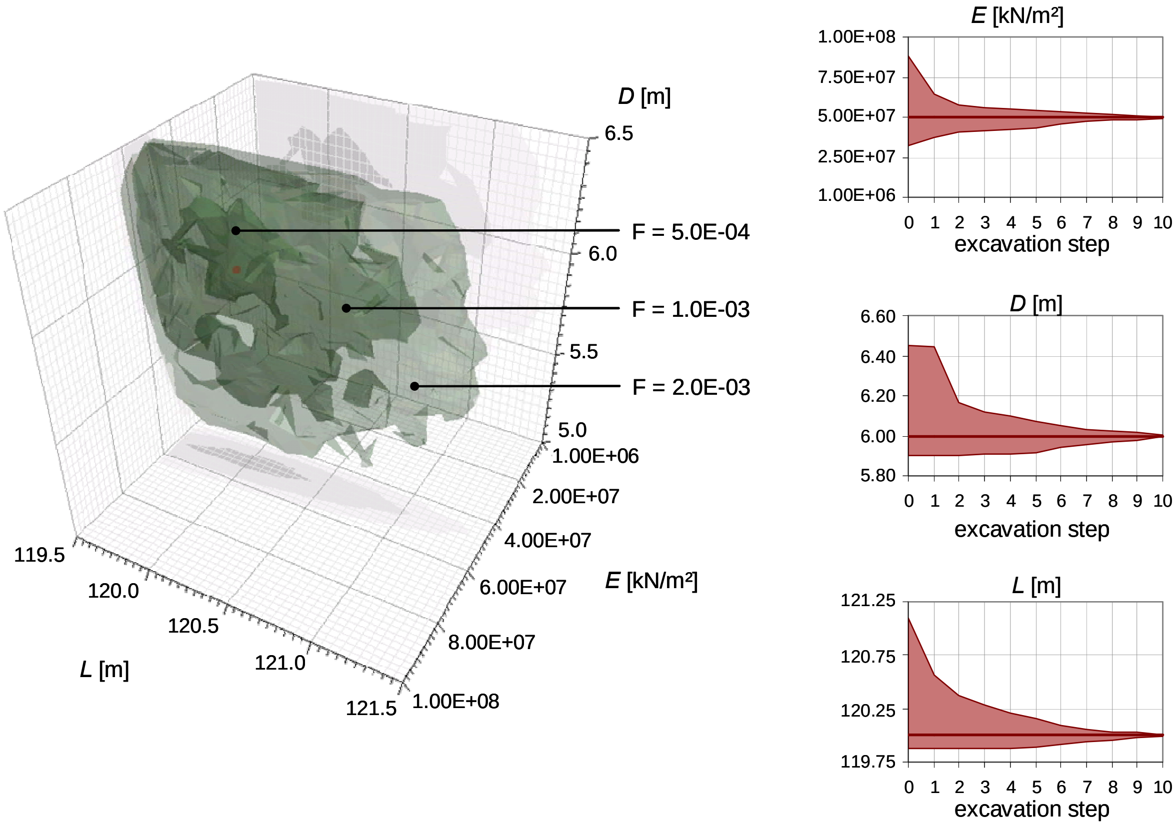

Figure 4: Left - 3D graph presenting the parameters sets giving the same value for the objective function. Right – development of the parameters search bands with the excavation steps.

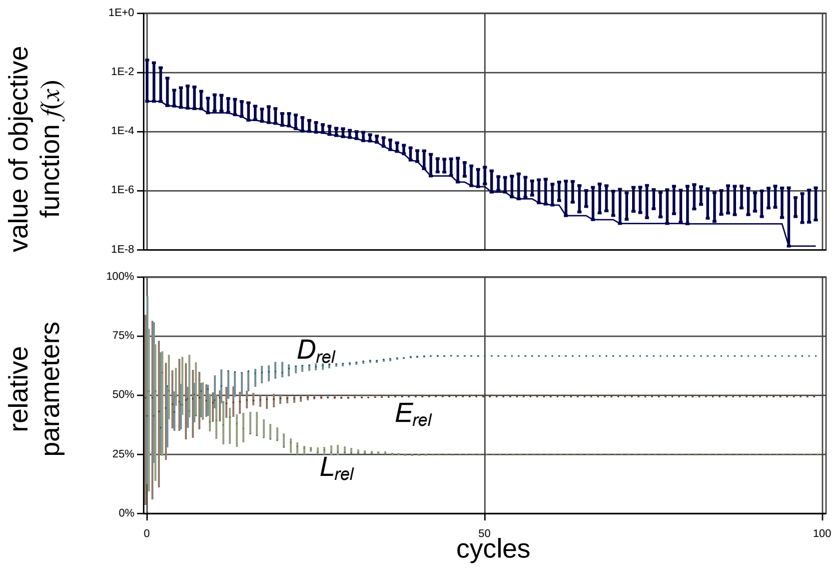

Figure 5: Objective function and relative parameters optimization range history plots.

REFERENCES

[1] E. Button, G. Riedmueller, W. Schubert, K. Klima, E. Medley: Tunnelling in tectonic melanges—accommodating the impacts of geomechanical complexities and anisotropic rock mass fabrics. *Bull Eng Geol Env*, 63 (2004),109–117.

[2] A. Cividini, L. Jurina, G. Gioda: Some aspects of characterization problems in geomechanics. *Int J Rock Mech Min Sci and Geomech Abstr*, 18(6) (1981), 487 – 503.

[3] R. C. Eberhart, J. Kennedy: A new optimizer using particle swarm theory. *Proceedings of the Sixth International Symposium on Micromachine and Human Science*, Nagoya, Japan, 1995, 30 – 43

[4] J. Kennedy, R. C. Eberhart: Particle swarm optimization. *Proceedings of the IEEE International Conference on Neural Networks*, Piscataway, NJ:IEEE Press,, 1995, 1942 – 1948

[5] G. Maier, L. Jurina, K. Podolak: On model identification problems in rock mechanics. *In: Proceedings of Symposium on the Geotechnics of Structurally Complex Formations, Capri*, (1997), 257–261.

[6] B. F. J. Manly: Multivariate statistical methods – a primer. 3rd Edition, Chapman & Hall / CRC, 1944.

[7] J. Meier, S. Rudolph, T. Schanz: Effektiver Algorithmus zur Lösung von inversen Aufgabenstellungen – Anwendung in der Geomechanik. *Bautechnik,* 83 (2006), Heft 7, 470 - 481.

[8] T. Schanz, M. M. Zimmerer, M. Datcheva, J. Meier: Identification of constitutive parameters for numerical models via inverse approach. *Felsbau*, 24 (2006), 11 - 21.

[9] W. Schubert, K. Grossauer, E. A. Button: Interpretation of displacement monitoring data for tunnels in heterogeneous masses. *Int. J. Rock Mech. Min. Sci.*, 41(6) (2004), CD-ROM.

[10] J. Will, D. Roos, J. Riedel, C. Bucher: Robustness Analysis in Stochastic Structural Mechanics. NAFEMS Seminar - Use of Stochastics in FEM Analyses, 2003, Wiesbaden